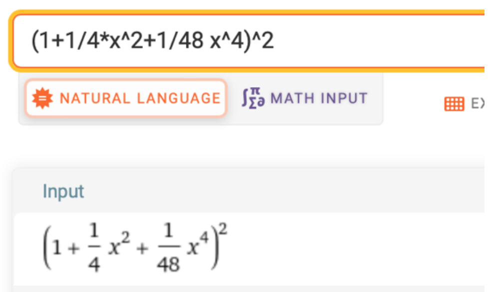
We will use Reduction of Order to find the second series solution to the last example we saw. Using the Method of Frobenius, we find that $y_1 = 1 + \frac{1}{4}x^2 + \frac{1}{48}x^4 + \frac{1}{1152}x^6 + \cdots$.

Recall that the Reduction of Order formula is $y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$. This is quite nasty by hand and can even be a challenge for computer algebra systems. So we break it down into steps.

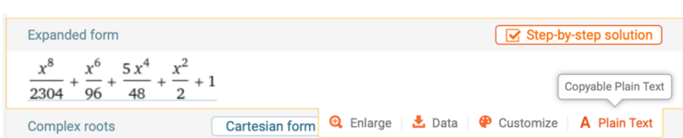
Since $P(x) = \frac{3}{x}$, we have $y_2 = y_1 \int \frac{1}{x^3\left(1+\frac{1}{4}x^2+\frac{1}{48}x^4+\frac{1}{1152}x^6+\cdots\right)^2}\,dx$.

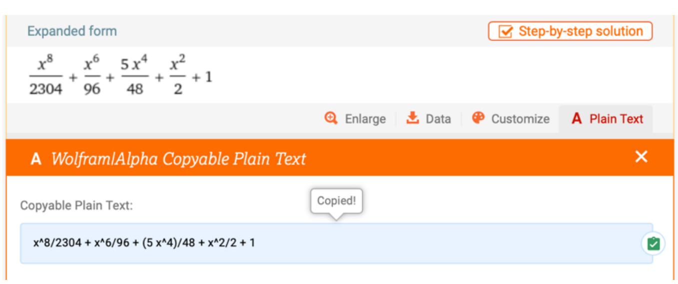
First, we deal with the y_1^2 factor in the denominator. Using WolframAlpha.com, follow these steps:

We only need a few terms for the final answer, so enter the following:

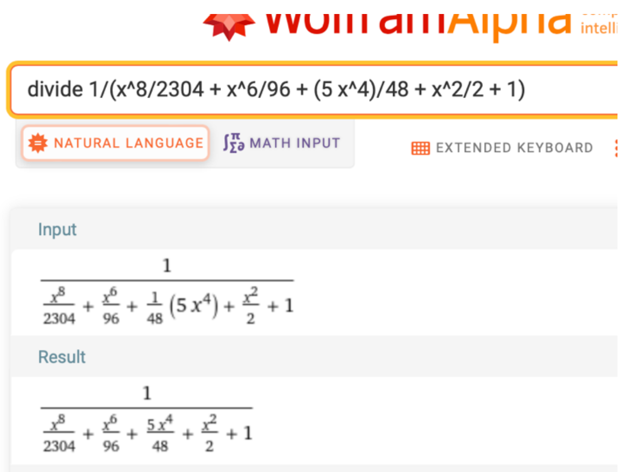


Scroll down to Expanded Form. Then if you hover over the output, you will see a button appear that says Plain Text. Click on that and then click on the line that says copyable text (click inside the white box that has the answer).



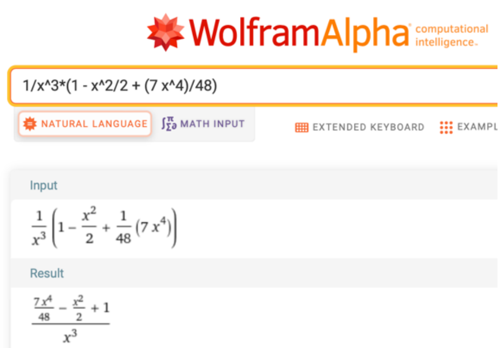


This is going into the denominator of the integrand, so we need to turn it into a fraction. Go back up to the input bar and enter a divide command with a numerator of 1 and then paste (inside parentheses) what you just copied.

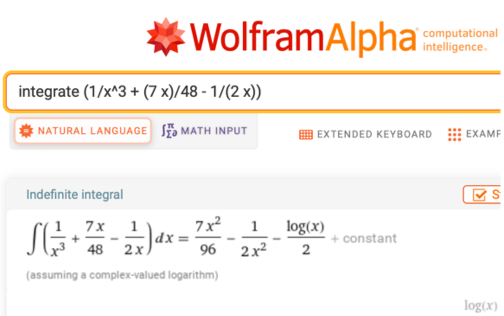


Scroll down again. This time you're looking for Series Expansion at $x = 0$. Use the Plain Text/copy feature to copy that output.

What we have now is $\frac{1}{y_1^2}$, but remember that the denominator of the integrand also has a factor of x^3. Thus we need to multiply $\frac{1}{x^3} \cdot 1/y_1^2$. In WolframAlpha, type $\frac{1}{x^3}$ and open a set of parentheses. Here you paste the series expansion. Delete everything after the third term. We'll talk about the $O(x^6)$ thing at another time.



Copy the result, and we now have our integrand. Using the integrate command, we obtain the integral part of y_2.



You'll see in my teaching notes in Canvas an acceptable way to present the solution.